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Extending the Theory of Arrays: `memset`, `memcpy`, and Beyond

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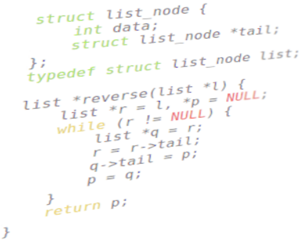

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- SMT-solvers are routinely used in program analysis:
  - Deductive program verification
  - Symbolic execution
  - Software bounded model checking
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  - ...
- Prominent theory: $\mathcal{T}_{\mathcal{A}}$ (theory of arrays)
  - Model arrays/structures/objects in the program
  - Model main memory

$\mathcal{T}_{\mathcal{A}}$: The Theory of Arrays

index terms	$t_\mathsf{I} ::= \ldots$
element terms	$t_\mathsf{E} ::= \ldots \mid \mathtt{read}(t_\mathsf{A}, t_\mathsf{I})$
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a write modifies the position written to ...

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... and nothing else

Motivation

How to model standard library functions such as `memset` and `memcpy`?

```
void *memset(void *dst, int c, size_t n);
```

```
void *memcpy(void *dst, const void *src, size_t n);
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might not be constant!

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Motivation

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```
...
memcpy(a, b, 4);
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Motivation

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$$a_1 = \texttt{write}(a, 0, \texttt{read}(b, 0))$$

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$$a_1 = \mathtt{write}(a, 0, \mathtt{read}(b, 0))$$

$$a_2 = \mathtt{write}(a_1, 1, \mathtt{read}(b, 1))$$

Motivation

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$$a_3 = \mathtt{write}(a_2, 2, \mathtt{read}(b, 2))$$

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$$a' = \mathtt{write}(a_3, 3, \mathtt{read}(b, 3))$$

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Does not scale well for large constants

Motivation

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```
...
memcpy(a, b, n);
...
```

Motivation

```
...
memcpy(a, b, n);
...
```

???

Motivation

```
...
memcpy(a, b, n);
...
```

$$a' = \text{copy}(a, 0, b, 0, n)$$

Motivation

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```
...
memcpy(a, b, n);
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```

$$a' = \lambda i.\, \mathsf{ITE}(0 \leq i < n,\ \mathtt{read}(b, i),\ \mathtt{read}(a, i))$$

Motivation

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$\Longrightarrow$ Extend $\mathcal{T}_{\mathcal{A}}$ by λ-terms that describe arrays

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Motivation

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```
int i, j, n = ...;
int *a = malloc(2 * n * sizeof(int));
for (i = 0; i < n; ++i) {
    a[i] = i + 1;
}
for (j = n; j < 2 * n; ++j) {
    a[j] = 2 * j;
}
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$$a' = \lambda i.\, \mathsf{ITE}(0 \leq i < n,\ i + 1,\ \mathtt{read}(a, i))$$
$$a'' = \lambda j.\, \mathsf{ITE}(n \leq j < 2 * n,\ 2 * j,\ \mathtt{read}(a', j))$$

Contributions

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1. $\mathcal{T}_{\lambda\mathcal{A}}$: an extension of $\mathcal{T}_{\mathcal{A}}$ with λ-terms

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2. Satisfiability checking for $\mathcal{T}_{\lambda\mathcal{A}}$

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β-reduction

$\texttt{write}(a, p, v)$ could be simulated using $\lambda i.\ \mathrm{ITE}(p = i,\ v,\ \texttt{read}(a, i))$

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- Attaching metadata to memory regions (allocated, de-allocated, . . .)

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- Loops can often be automatically transformed into loops that satisfy these requirements

Satisfiability Checking

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- Two quantifier-free approaches
  - Eager reduction
  - Instantiation-based approach

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- Does not provide a decision procedure in general

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Evaluation

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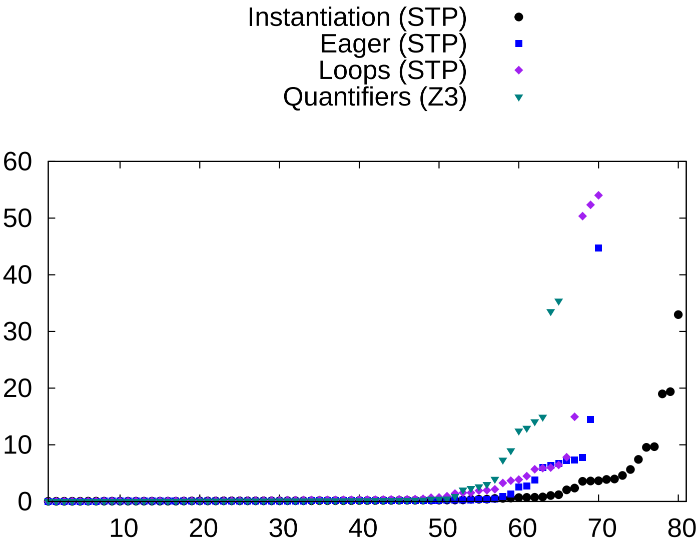
- Evaluated three reductions and loop unrolling
  - Quantifier-based approach using `Z3` and `CVC4`
  - Eager reduction and instantiation-based approach using `STP`, `Boolector`, `Z3`, and `CVC4`
  - Loop unrolling approach using `STP`, `Boolector`, `Z3`, and `CVC4`

Results

SMT solver	Approach	Total Time	# Solved Formulas	# Timeouts	# Aborts
STP	Instantiation	206.034	80	1	–
STP	Eager	779.544	70	11	–
STP	Loops	670.526	70	6	5
Boolector	Instantiation	818.782	71	10	–
Boolector	Eager	986.751	70	11	–
Boolector	Loops	1139.483	61	15	5
Z3	Instantiation	948.365	67	13	1
Z3	Eager	1043.632	66	15	–
Z3	Quantifiers	1122.489	65	16	–
Z3	Loops	1619.583	53	23	5
CVC4	Instantiation	928.079	67	14	–
CVC4	Eager	1119.748	65	16	–
CVC4	Quantifiers	1407.118	54	21	6
CVC4	Loops	1552.698	56	19	6

Results



Instantiation (STP)
Eager (STP)
Loops (STP)
Quantifiers (Z3)
60
50
40
30
20
10
0
10
20
30
40
50
60
70
80

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- Integration into an SMT-solver using "Lemmas-on-demand"/"lazy instantiation" is the next step

`http://llbmc.org`