SyMT: finding symmetries in SMT formulas
(Work in progress)

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Outline

Introduction

Satisfiability solving:

- problem encoding of primal importance
- doing many times the same thing is a waste of time

Previously (CADE 2011):

- breaking symmetries on QF_UF gives impressive results

In this talk:

- beyond tailored heuristics
- generalize symmetry finding

Outline

Symmetry breaking: break a factorial

- 4 distinct pigeons:
  $p_1 \neq p_2 \wedge p_1 \neq p_3 \wedge p_1 \neq p_4 \wedge$
  $p_2 \neq p_3 \wedge p_2 \neq p_4 \wedge p_3 \neq p_4$
- every pigeon in a hole:
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- large decrease of search space, large decrease in solving times

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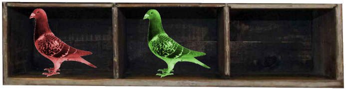

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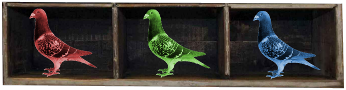

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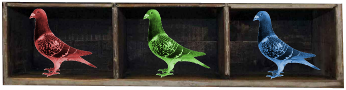

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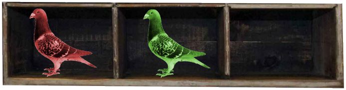

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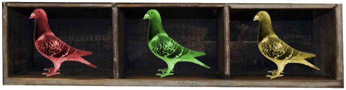

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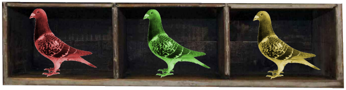

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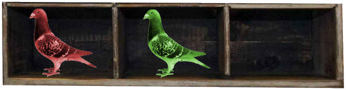

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Previous technique

The formula:

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- B_1, B_2, B_3 appear the same number of times, in the same number of clauses,...
- formula preserved by permutation of B_1, B_2, B_3 ($\vee$, $\wedge$ commutative)
- $p_1 = B_1 \vee p_1 = B_2 \vee p_1 = B_3$ holds, so let's say $p_1 = B_1$
- resulting formula is still symmetric w.r.t. B_2, B_3. Repeat

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Previous technique: weaknesses

Besides being sensitive to obfuscation:

- finding symmetries highly heuristic: guess, and then check symmetry
- symmetry breaking tailored to special case: not easily generalizable

But impressive improvements on QF_UF: worth trying to extend

Goal: being more general in finding and breaking symmetries

Outline

Finding symmetries: graph isomorphism

Graph isomorphism problem

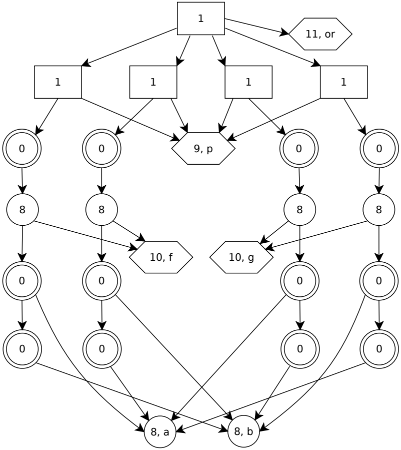
Finding non trivial renaming of nodes resulting in isomorphic graph

- graph isomorphism finding is in NP. P? NPC?
- efficient algorithms exist (time never an issue in our experiments)
- formulas: not exactly graphs, but can easily be translated
- btw, isomorphism finding for DAGs is not simpler
- good tools: saucy, bliss

Finding symmetries: from formulas to (acyclic) graphs

$$p(f(a,b)) \vee p(f(b,a)) \vee p(g(a,b)) \vee p(g(b,a))$$

- node colored by (HOL) sort
- node for every symbol
- node for each subterm
- special nodes "0" for lists of terms
- node for term linked to top symbol and list of subterms
- commutative symbols: directly link to subterms
- maximal sharing



Graph isomorphic iff formula symmetric

Finding symmetries: the tool

- parse formula
- simplify (several options)
- graph isomorphism tool: saucy, bliss
- output group generators

```
./SyMT -enable-simp smt-lib2/QF_UF/NEQ/NEQ004_size4.smt2
    (p7 p9)(c12 c13)
    (c_3 c_1)
    (c_2 c_1)
    (c_0 c_1)
```

SMT-LIB and symmetries

Category	#Inst	#Sym[P]	Avg[GS]	Time
AUFLIA	6480	6258	134.00	378.79
AUFLIRA	19917	16476	1.08	9.13
AUFNIRA	989	985	1.00	0.41
QF_AUFLIA	1140	78	1.00	0.72
QF_AX	551	22	1.00	0.37
QF_IDL	1749	756	12745.43	327.95
QF_LIA	5938	1200	104.55	486.19
QF_LRA	634	210	110.49	29.06
QF_NIA	530	169	5.92	3.92
QF_NRA	166	43	1.00	0.23
QF_RDL	204	24	0.00	10.13
QF_UF	6639	3638	44.00	34.58
QF_UFIDL	431	189	1.00	2.70
QF_UFLIA	564	198	0.00	0.45
UFNIA	1796	1070	47.08	543.26

Outline

Teaser: symmetry simplification

Issues with symmetries:

- graph isomorphism tools provide (small number of) generators
- maybe redundant even if…
- …some kind of non redundancy property holds
- figuring out what generators mean can be difficult

Work in progress:

- simplify generators
- identify subgroups that are full permutation groups on some symbols
- computational group theory: Schreier-Sims (polynomial)

E.g.

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```

Teaser: symmetry breaking

Symmetry breaking for propositional logic? Set of formulas:

$$\psi_{i,\sigma} =_{\text{def}} \Big(\bigwedge_{1 \le j < i} p_j \equiv p_j\sigma \Big) \Rightarrow (p_i \Rightarrow p_i\sigma).$$

assuming an order on propositional variables

- SBP on SAT: large, need advanced techniques
- working on recasting to SMT
- SMT symmetries are more “structural”
- hopefully easier to break efficiently (?)
- Symmetry breaking for SMT unifies several heuristics, e.g. diamonds

Outline

Conclusion

- symmetry-based techniques sensitive to obfuscation
- **users should break their symmetries themselves**, i.e. generate symmetry-free formulas
- SyMT, a tool to find out symmetries
- in the near future, SyMT will provide hints for symmetry breaking predicates
- open-source (w.i.p.). `http://www.veriT-solver.org`